

問1：次の微分方程式をラプラス変換を用いて解け。

$$\frac{dx(t)}{dt} + 3x(t) = I(t), \quad x(0) = 0.0$$

解答：

$$sX(s) + 3X(s) = \frac{1}{s}$$

$$A = sX(s)\Big|_{s=0} = \frac{1}{s+3}\Big|_{s=0} = \frac{1}{3}$$

$$(s+3)X(s) = \frac{1}{s}$$

$$B = (s+3)X(s)\Big|_{s=-3} = \frac{1}{s}\Big|_{s=-3} = -\frac{1}{3}$$

$$X(s) = \frac{1}{s(s+3)}$$

$$X(s) = \frac{1}{3}\left(\frac{1}{s} - \frac{1}{s+3}\right)$$

$$X(s) = \frac{A}{s} + \frac{B}{s+3}$$

$$\therefore x(t) = \frac{1}{3}(I(t) - \exp(-3t))$$

問2：次の微分方程式をラプラス変換を用いて解け。

$$\frac{d^2x(t)}{dt^2} + 5\frac{dx(t)}{dt} + 6x(t) = 0, \quad x(0) = 1.0, \quad dx(0)/dt = 0.0$$

解答：

$$s^2X(s) - s + 5sX(s) - 5 + 6X(s) = 0$$

$$X(s) = \frac{s+5}{(s+2)(s+3)} = \frac{A}{s+2} + \frac{B}{s+3}$$

$$(s^2 + 5s + 6)X(s) = s + 5$$

$$A = (s+2)X(s)\Big|_{s=-2} = \frac{3}{1}$$

$$X(s) = \frac{s+5}{(s+2)(s+3)}$$

$$B = (s+3)X(s)\Big|_{s=-3} = \frac{2}{-1} = -2$$

$$X(s) = 3\frac{1}{s+2} - 2\frac{1}{s+3}$$

$$\therefore x(t) = 3\exp(-2t) - 2\exp(-3t)$$